Week 15 Worksheet - Last Worksheet

Instructions. Follow the instructions given by your TA. You are not expected to finish all the problems. :)

1. (a) Compute $\frac{d}{dx}\int_{\ln x}^{x^3} e^{\sin t}dt$

   (b) $\lim_{x\to 0}\frac{\int_1^{1+5x}(4-\cos 2\pi t)^3 dt}{x}$

   (c) $\lim_{r\to 0}\frac{1}{r}\int_{e^{4r}}^{1}\sqrt{3+\frac{1}{x}}dx$

   (Remark: Other ways of testing could be asking max/min, increasing interval, concave etc)

(a) $? = 3x^2 e^{\sin(x^3)} - \frac{1}{x}e^{\sin(\ln x)}$

(b) "$\frac{0}{0}$" L'Hopital

$? = \lim_{x\to 0} \frac{5\left[4-\cos(2\pi(1+5x))\right]^3}{1} = 5(4-\cos 2\pi)^3 = 5\cdot 3^3$

(c) "$\frac{0}{0}$" L'Hopital

$? = \lim_{r\to 0}\frac{-4e^{4r}\sqrt{3+\frac{1}{e^{4r}}}}{1}$

$= -4\sqrt{3+1}$

$= -8$

2. *(from 2016 final)*

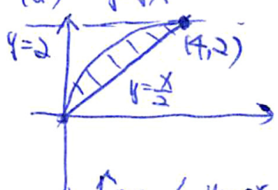
   Write down the integral which represent the volumes of the following solids. (no need to compute them)

   (a) The solid generated by revolving the region in the 1st quadrant bounded by $y=\sqrt{x}$, $y=\frac{x}{2}$ about the line $y=2$.

   (b) The solid generated by revolving the region in the 1st quadrant bounded below by $y=e^x$ and above by $y=e$, about the y−axis.

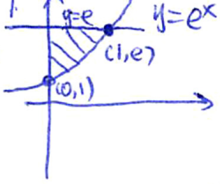
   (c) The solid generated by revolving the region in the 1st quadrant bounded to the right by $y=\sqrt{4-x^2}$ and to the left by $y=x$ about the line $x=-1$.

(a)



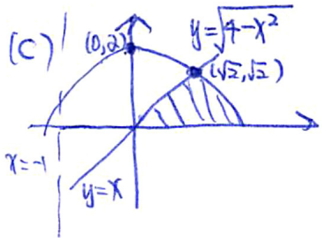
$\int_0^4 \pi(2-\frac{x}{2})^2 - \pi(2-\sqrt{x})^2\,dx$

(b)



$\int_1^e \pi(\ln y)^2\,dy$

(c)



$\sqrt{4-x^2}=x \Rightarrow 4-x^2=x^2 \Rightarrow x=\sqrt{2}$ $(\sqrt{2},\sqrt{2})$ is the intersection pt.

$\int_0^{\sqrt{2}} \pi(\sqrt{4-y^2}+1)^2-\pi(y+1)^2\,dy$

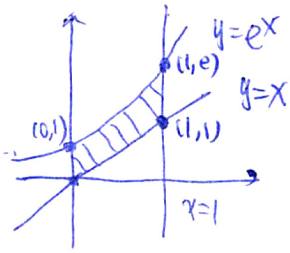
3. *(from 2015 final)*

Consider the region in the first quadrant bounded above by the function $y = e^x$, below by $y = x$, to the left by $y-$axis, and to the right by $x = 1$.

(a) Set up an integral that describes the area of the region. (no need to calculate the integral)

(b) Set up an integral describing the volume of the solid generated by the rotating around the $x-$axis.

(a) $\int_0^1 e^x - x \quad dx$

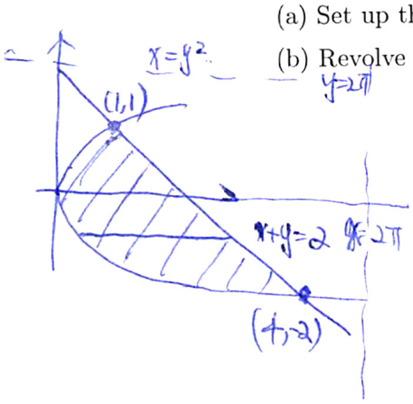


(b) $\int_0^1 \pi(e^x)^2 - \pi(x)^2 dx$

4. Consider the finite region bounded by $x = y^2$ and $x + y = 2$.

(a) Set up the integral for its area

(b) Revolve the region around $\overset{x}{\cancel{y}} = 2\pi$. Set up the integral for the volume.



(a) intersection pt.

$y^2 = 2-y \Rightarrow y^2+y-2=0 \quad (y+2)(y-1)=0 \quad y=-2$ or $y=1$

2 intersection pts: $(4,-2)$ $(1,1)$

$\int_{-2}^{1} 2-y - y^2 \quad dy$

around $x=2\pi$

(b) $\int_{-2}^{1} \pi(2\pi - y^2)^2 - \pi(2\pi - 2+y)^2 dy$

around $y=2\pi$

$\int_{-2}^{1} 2\pi(2\pi-y)(2-y-y^2)\,dy$